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DETERMINATION OF TIME CONSTANT OF LOW-INERTIA RESISTANCE THERMOMETERS BY THE ALTERNATING CURRENT METHOD

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DETERMINATION OF TIME CONSTANT OF LOW-INERTIA RESISTANCE THERMOMETERS BY THE ALTERNATING CURRENT METHOD

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Abstract: A method is analyzed for determining the time constant of a resistance thermometer by double heating with direct and alternating currents.

The time-averaged temperature of a conductor heated by alternating current with a non-zero temperature coefficient of resistance depends on the form, frequency and magnitude of the current and the time constant of the wire. The heating of conductors by sine-wave current was analyzed in [1].

In order to determine the time constant of a conductor, it is convenient to use alternating current in the form of Π-shaped pulses.

The idea of determining time constant ε of a resistance thermometer by the alternating current method is that at first the thermometer is heated by constant current I to temperature θ, then under unchanged circumstances of heat exchange it is heated by alternating current with time-averaged value I_{av} to the average temperature t_{av} such that $t_{av} = \theta$. Constant ε is determined by the ratio of currents I_{av}/I.

Let us study a conductor (thermometer) with non-zero temperature coefficient of resistance. The conductor has resistance R and is placed in a stream of gas with constant temperature and speed (Figure 1). Thermometer R is connected into a double bridge circuit consisting of resistors with high thermal inertia. Power supply to the bridge may be either by dc or by a current in the form of Π-shaped pulses, which can be achieved in many cases using a simple generator as shown on Figure 1.

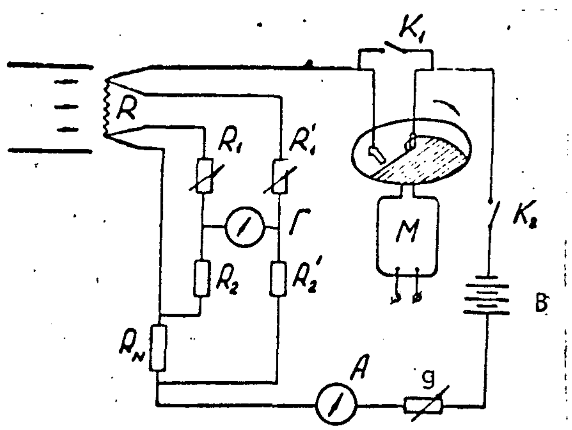

Figure 1. Diagram of Determination of Time Constant

As the thermometer is heated by direct current I, its temperature θ, calculated from the temperature of the gas, remains unchanged with time

$$\vartheta = \frac{I^2 R}{\alpha F}, \tag{1}$$

where α is the heat transfer factor on the surface of the thermometer; F is the heat-liberating surface.

When the same thermometer is fed by current in Π-shaped pulses with a pulse length a and pulse repetition period b, heating and cooling processes will alternate.

With proper selection of a and b, we can produce a heating and cooling curve for the thermometer and shown schematically on Figure 2. The temperature of the thermometer t will oscillate about average value t_{av}.

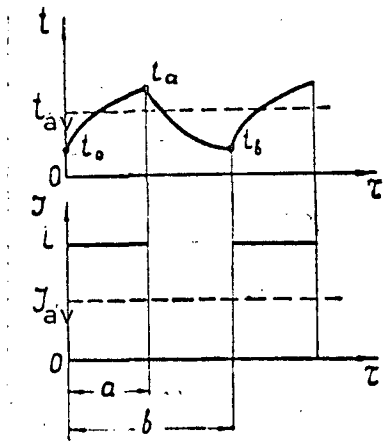

Figure 2. Nature of Change of Current and Temperature

We consider that the changes in t are slight, and therefore assume a linear dependence between t and R.

$$R = R_0[1 + \beta(t - t_{av})], \tag{2}$$

where R_0 and β are the resistance and temperature coefficient of resistance at temperature t_{av}. We assume also that heat transfer through the ends of the thermometer is negligible and that there are no temperature gradients over the cross section of the thermometer. Then the heat balance equation can be written as:

$$\varepsilon \frac{dt}{d\tau} + t(t - k\beta) = k - k\beta t_{av}, \tag{3}$$

where

$$k = \frac{i^2 R_{av}}{aF}. \tag{4}$$

Here ε is the time constant;
 k is the heating factor;
 i is the current during a pulse;
 τ is time.

Considering that at the initial moment in time of the heating process, for which $\tau = 0$, the temperature of the thermometer was t_0, we produce [2]:

$$t = \left(t_0 - \frac{k - k\beta t_{av}}{1 - k\beta}\right) e^{-\frac{1}{\varepsilon}(1 - k\beta)\tau} + \frac{k - k\beta t_{av}}{1 - k\beta}. \tag{5}$$

For the cooling sector with current i = 0, equation (3) is simplified to

$$t = t_a e^{-\frac{1}{\varepsilon}(\tau - a)}, \tag{6}$$

where t_a is the temperature of the thermometer at the moment of the beginning of cooling.

During the process of repeated heating and cooling of the thermometer, unchanged temperatures are established at the beginning of each interval of heating t_0, the end of each interval of heating and the beginning of each interval of cooling t_a and the end of each interval of cooling t_b. Obviously, $t_0 = t_b$. Using these conditions, we find

$$t_0 = \frac{k - k\beta t_{av}}{1 - k\beta} \frac{\left[1 - e^{-\frac{1}{\varepsilon}(1 - k\beta) a}\right] e^{-\frac{1}{\varepsilon}(b - a)}}{1 - e^{-\frac{1}{\varepsilon}[(1 - k\beta) a + (b - a)]}}, \tag{7}$$

$$t_a = \frac{k - k\beta t_{av}}{1 - k\beta} \frac{1 - e^{-\frac{1}{\varepsilon}(1 - k\beta) a}}{1 - e^{-\frac{1}{\varepsilon}[(1 - k\beta) a + (b - a)]}}. \tag{8}$$

In the area of the equilibrium state of the bridge, its characteristics are linear, the current in its diagonals is proportional to R. Therefore, if the time-averaged value of current in the diagonals of the bridge is equal to zero, we can use the known resistance of its comparison arms to find the time-averaged resistance R_{av}.

The unknown resistance through the bridge circuit is determined only during a current pulse, since during pauses the current in the diagonals is always equal to zero; therefore, the value of R_{av} determined corresponds to time-averaged temperature t_{av} of the thermometer during the process of heating.

In order to find t_{av}, we substitute t_0 from (7) into (5), multiply it by $d\tau$, integrate between 0 and a and divide by a. The equation produced can be solved for t_{av}:

$$t_{av} = k \frac{(1 - k\beta) - \frac{\Psi m}{\gamma}}{(1 - k\beta) - \frac{\Psi m}{\gamma} k\beta}. \tag{9}$$

here

$$m = \frac{\left[1 - e^{-\frac{1}{\Psi}(1-\gamma)}\right]\left[1 - e^{-\frac{1}{\Psi}(1-k\beta)\gamma}\right]}{1 - e^{-\frac{1}{\Psi}(1-k\beta)\gamma}}, \tag{10}$$

Ψ is the relative time constant, $\Psi = \varepsilon/b$; γ is the pulse duty factor, $\gamma = a/b$.

If the thermometer is fed with a direct current I and the bridge is balanced, we can find the resistance of the thermometer R and its temperature θ. Then the source of direct current is turned off and the thermometer is fed with the Π-shaped current pulses with time-averaged value $I_{av} = i/\gamma$ and period b with unchanged heat exchange coefficients between the thermometer and the gas medium. Current I_{av} is selected so that $\theta = t_{av}$, which occurs with the same values of bridge arm resistances as before and with zero time-averaged current in the diagonals.

Then, equating (1) and (9), we find

$$\bar{i} = \frac{I_{av}}{I} = \frac{\gamma}{\sqrt{\dfrac{(1-k\beta) - \frac{\Psi m}{\gamma}}{(1-k\beta) - \frac{\Psi m}{\gamma} k\beta}}}. \tag{11}$$

Determination of ratio $\bar{i}$ does not require measuring the currents directly flowing through the thermometer. With unchanged resistances of the arms of the bridge and with the bridge in balance, the supply current of the entire bridge is proportional to the thermometer current; therefore, $\bar{i}$ is equal to the ratio of currents flowing through the entire bridge.

In order to determine $k\beta$, we find αF from (1), substitute it into (4), after which we produce;

$$k\beta = \left(\frac{I_{av}}{I\gamma}\right)^2 \theta\beta. \tag{12}$$

As a result of this experiment we find $\bar{i} = I_{av}/I$, after which we use (12) to calculate $k\beta$ and, using the table of functions of $\bar{i}$ produced from (11), we find Ψ. The time constant

$$\varepsilon = b\Psi. \tag{13}$$

In order for the values of ε produced by this method to be of acceptable accuracy, certain conditions must be fulfilled.

Determination of ε is impossible if the frequency of the supply current is too low or too high. There is a certain value of period b for which the greatest accuracy can be achieved. Let us study this problem for a pulse duty factor $\gamma = 0.5$.

This method requires determination of Ψ on the basis of current ratio $\bar{i}$, determined experimentally; this ratio, of course, always contains a certain error. The error in $\bar{i}$ will have minimum influence on error in Ψ when $d\bar{i}/d\Psi$ has its greatest value. Therefore, current period b should be selected such that $d\bar{i}/d\Psi$ is maximal. Figure 3 shows a graph of function $\bar{i}$ for the significant area of values of Ψ. Function $\bar{i}$ changes little as $k\beta$ changes; the values of $k\beta$ will generally fall in the interval $0.05 < k\beta < 0.35$. Therefore, let us study the average value $k\beta = 0.2$. For $k\beta = 0.2$, the dotted line on Figure 3 shows us $d\bar{i}/d\Psi$, which has a maximum at $\Psi \approx 0.1$. Therefore, the period of pulse repetition should be selected from the condition $b \approx 10\varepsilon$, for which $\bar{i} \approx 0.56$.

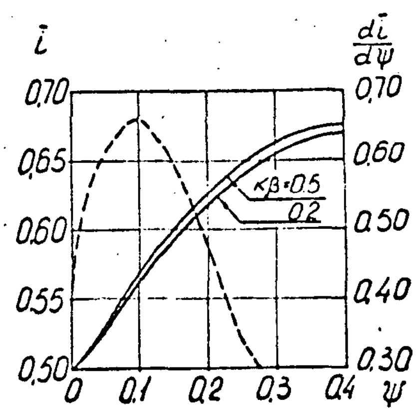

Figure 3. Functions $\bar{i}$ and $d\bar{i}/d\Psi$ for $\gamma = 0.5$ in the Working Area of Ψ

Let us perform an estimate of the error in determination of ε for the optimal period b and $k\beta = 0.2$. In the area $\Psi = 0.1$ and $k\beta = 0.2$, according to (10), m changes little and is near 1, so that for purposes of an approximate estimation we can assume $m = 1$.

Then from (11) and (12) we find:

$$d\Psi = \frac{1}{A^2}\Big\{2\bar{i}\gamma C(A-B)\,d\bar{i} + C[(\bar{i}^2 - 3\gamma^2)A + 2B\gamma^2 k\beta]\,d\gamma + B\gamma(C\gamma^2 - A)\,d\,k\beta\Big\}, \tag{14}$$

$$d\,k\beta = \frac{1}{\gamma^2}\Big(2\vartheta\beta\bar{i}\,d\bar{i} - \frac{1}{\gamma}2\bar{i}^2\vartheta\beta\,d\gamma + \bar{i}^2\beta\,d\vartheta\Big), \tag{15}$$

where $A = \overline{i}^2 - \gamma^2 k\beta$; $B = \overline{i}^2 - \gamma^2$; $C = 1 - k\beta$.

After substituting the numerical values, we produce expressions for estimation of the absolute errors Δ:

$$|\Delta\Psi| \leqslant |1{,}3 d\overline{i}| + |-1{,}25 d\gamma| + |-0{,}03 dk\beta|,$$

$$|\Delta k\beta| \leqslant |1{,}8 d\overline{i}| + |-2{,}0 d\gamma| + |0{,}005 d\vartheta|.$$

Table

Symbol	Quantity	$\pm\delta$, %	$\pm\Delta$	Symbol	Quantity	$\pm\delta$ %	$\pm\Delta$
I	432 ма	0,2	0,860 ма	$k\beta$	0,20	7,0	0,015
I_{av}	242 ма	0,2	0,485 ма	Ψ	0,100	6,4	0,0064
$\overline{i}$	0,5608	0,4	0,0022	b	78,9 msec	1,0	0,79 msec
γ	0,50	0,5	0,0025	ε	7,89 msec	7,4	0,58 msec

As an example, the table presents the numerical values of the limiting absolute errors Δ and relative errors δ of values produced in a determination of the time constant of a tungsten resistance thermometer 25 microns in diameter, over which a current of air was blown. The limiting relative error ε does not exceed ±7.5%. The accuracy of determination of ε in these experiments was limited primarily by errors in the pulse duty factor. Use of a higher quality generator can provide higher accuracy of determination of ε.

Measurement of currents using this method should be performed with magnetoelectric devices containing integrators and showing the time-averaged current value. They can operate normally if their internal time constant is at least an order of magnitude higher than the pulse period. This fact should also be considered in selecting the frequency of the supply current.

BIBLIOGRAPHY

1. Pelepeychenko, I. P., Simbirskiy, D. F., "Effect of Displacement of Average Temperature Level in Heating of Conductors By Alternating Current," *Elektrichestvo*, No 3, 1966.

2. Kamke, E., *Spravochnik po Obyknovennym Differentsial'nym Uravneniyam* [Handbook on Ordinary Differential Equations], Moscow, Nauka Press, 1965.

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